University of KwaZulu-Natal

Discipline of Electrical, Electronic and Computer Engineering

ENEL 3SSH1 Systems and Simulations

Main Examination 2026

Duration: 2 hours. **Maximum mark: 100.** **Date: 26 May 2026**

Examiner: Prof. J. T. Agee **Moderator: Dr. S. S. Patel**

Instructions: *This question paper has four question, spread over two pages. Answer all four questions. A two-page formula sheet is attached at the end of the paper.*

You may write your answers in blue or black ink. Do not write in pencil or any colour of ink other than the two colours advised, otherwise such work would not be marked.

Plots not made using the appropriate graph sheets will not be marked.

No programmable calculators allowed during this examination.

QUESTION ONE [26 Marks]

An RLC circuit with three current loops is shown in Figure 1. The parameters R_1, R_2, R_3 and R_4 are resistors. L_1 is an inductor, and C_1, C_2 are capacitors. The current in Loop 1 is $i_1(t)$. The currents $i_2(t)$ and $i_3(t)$ flow in Loop 2 and Loop 3 respectively.

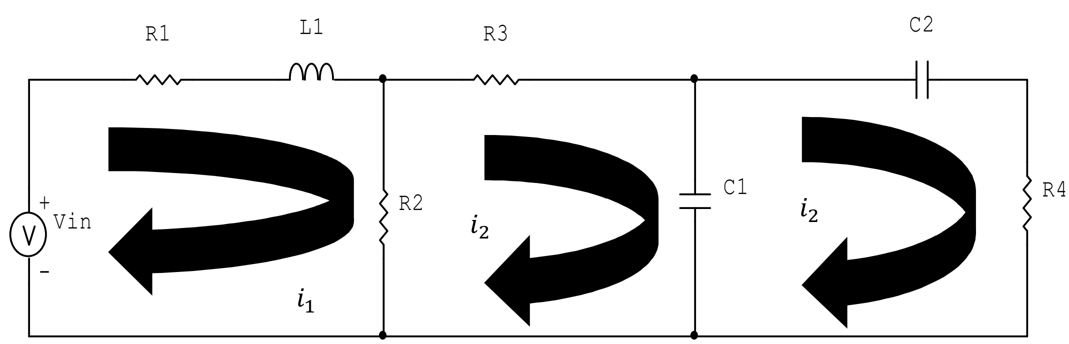

Figure 1: An RLC circuit containing two capacitors and one inductor

1.1 Derive the dynamic equations for the RLC circuit. [7]

1.2 Using the state assignment $x^T = [i_1\ V_{c1}\ \ V_{C2}]$,$y = i_3$ and the input $u = V_{in}$, express the model of the RLC circuit in the state-space form. [19]

QUESTION Two [37 Marks]

You are required to convert the third-order control system having the transfer function

$G_1(s) = \frac{s^2+8s+5}{s^3+17s^2+92s+160}$ into equivalent state-space representations:

2.1 Derive the observability canonical state-space representation of $G_1(s)$. [12]

2.2 Derive the Cascade state-space representation of the system. [13]

2.3 Derive the parallel state-space representation of the system. [12]

QUESTION THREE [22 Marks]

3.1 Derive the impulse response of the system $G_2(s) = \frac{s^2+2s+8}{s^3+9.5s^2+23.5s+15}$. [10]

3.2 Derive the step response of the system $G_3(s) = \frac{1}{s^2+2s+4}$. [12]

QUESTION FOUR [15 Marks]

The dynamic of the nonlinear system given by $\frac{dy}{dt} = y + t + ty$ is to be simulated numerically. Given the system's initial condition $t_0 = 0; y_0 = 1$, and the simulation interval of h=0.05 sec, use the Euler method to obtain the value of y when t=0.15 sec.

Properties of Laplace Transform

Given: $\mathscr{L}\{f(t)\} = F(s)$

	t-domain	s-domain
1.	$1(t-a)f(t-a)$	$e^{-as}F(s)$
2.	$e^{-\alpha t}f(t)$	$F(s+\alpha)$
3.	$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}\{F(s)\}$ $n \geq 0$, n integer
4.	$\frac{f(t)}{t}$	$\int_s^\infty F(s)\,ds$
5.	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
6. (a)	$\frac{df(t)}{dt}$	$sF(s) - f(0^+)$
(b)	$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1}f(0^+) \cdots - f^{(n-1)}(0^+)$
7.	$\int_{-\infty}^{t} f(\tau)\,d\tau$	$\frac{F(s)}{s} + \frac{f^{(-1)}(0^+)}{s}$
8.	$\int_0^t f_1(\tau)f_2(t-\tau)\,d\tau$	$F_1(s)F_2(s)$
9.	$f(t) = f(t+T)$, T = period	$\frac{\int_0^T f(t)e^{-st}\,dt}{1-e^{-sT}}$
10.	$\lim_{t\to 0} f(t)$	$\lim_{s\to\infty} sF(s)$
11.	$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0} sF(s)$, if $sF(s)$ is analytic in the right half s-plane
12.	$f_1(t)f_2(t)$	$\frac{1}{2\pi j}\int_{\sigma_1-j\infty}^{\sigma_1+j\infty} F_1(p)F_2(s-p)\,dp$ $\sigma > \sigma_1 + \sigma_{II}$ $\sigma_1 < \sigma_1 < \sigma - \sigma_{II}$

LAPLACE TRANSFORMS

Table 2.2-2 Useful Laplace Transforms

$f(t)$	$F(s)$
1. $\delta(t)$	1
2. 1	$\frac{1}{s}$
3. t^n	$\frac{\Gamma(n+1)}{s^{n+1}}$, Γ = Gamma function. $n > -1$ $\Gamma(n+1) = n!$ for n an integer
4. $e^{-\alpha t}$	$\frac{1}{s+\alpha}$
5. $\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$
6. $\cos \beta t$	$\frac{s}{s^2+\beta^2}$
7. $\sinh \beta\tau$	$\frac{\beta}{s^2-\beta^2}$
8. $\cosh \beta t$	$\frac{s}{s^2-\beta^2}$
9. $e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$
10. $e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$
11. $\frac{t^n e^{-\alpha t}}{n!}$	$\frac{1}{(s+\alpha)^{n+1}}$, $n \geq 0$, integer
12. $t \sin \beta t$	$\frac{2\beta s}{(s^2+\beta^2)^2}$
13. $t \cos \beta t$	$\frac{s^2-\beta^2}{(s^2+\beta^2)^2}$
14. $\int_0^t \frac{\sin a\tau}{\tau}\,d\tau$	$\frac{1}{s}\tan^{-1}\frac{a}{s}$
15. $te^{-\alpha t}\sin \beta t$	$\frac{2\beta(s+\alpha)}{[(s+\alpha)^2+\beta^2]^2}$
16. $te^{-\alpha t}\cos \beta t$	$\frac{(s+\alpha)^2-\beta^2}{[(s+\alpha)^2+\beta^2]^2}$

[a]For the derivation of these properties, see S. C. Gupta, *Transform and State Varial*

Related Laplace, and Z transforms — Transients and Transfer Functions

$\mathcal{Z}\{x(iT)\}$	$x(t)$	$\mathcal{L}\{x(t)\} \big/ P_s(s)$	$P_z(z)=\frac{z-1}{z}\mathcal{Z}\left\{\left[\mathcal{L}^{-1}\left\{\frac{P_s(s)}{s}\right\}\right]_{t=iT}\right\}$
not defined	$\delta(t)$	1	1
1	any $f(t)$ with $f(iT)=\begin{cases}1, & i=0\\ 0, & i\neq 0\end{cases}$	not defined	not defined
$\frac{z}{z-1}$	1	$\frac{1}{s}$	$\frac{T}{z-1}$
$\frac{Tz}{(z-1)^2}$	t	$\frac{1}{s^2}$	$\frac{T^2(z+1)}{2(z-1)^2}$
$\frac{z}{z-e^{-aT}}$	e^{-at}	$\frac{1}{s+a}$	$\frac{1-e^{-aT}}{\left[z-e^{-aT}\right]a}$
$\frac{Tze^{-aT}}{\left(z-e^{-aT}\right)^2}$	te^{-at}	$\frac{1}{(s+a)^2}$	$\frac{\left[1-e^{-aT}(1+aT)\right]z+\left[e^{-aT}\left(e^{-aT}-1+aT\right)\right]}{\left[z-e^{-aT}\right]^2a^2}$
$\frac{ze^{-aT}\sin(bT)/b}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$	$e^{-at}\frac{\sin(bt)}{b}$	$\frac{1}{s^2+2as+a^2+b^2}$	$\frac{\left[1-e^{-aT}\left(\cos(bT)+\frac{a}{b}\sin(bT)\right)\right]z+\left[e^{-aT}\left(e^{-aT}-\cos(bT)+\frac{a}{b}\sin(bT)\right)\right]}{\left[z^2-2ze^{-aT}\cos(bT)+e^{-2aT}\right]\left(a^2+b^2\right)}$
$\frac{z^2-ze^{-aT}\cos(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$	$e^{-at}\cos(bt)$	$\frac{s+a}{s^2+2as+a^2+b^2}$	$\frac{\left[1-e^{-aT}\left(\cos(bT)-\frac{b}{a}\sin(bT)\right)\right]z+\left[e^{-aT}\left(e^{-aT}-\cos(bT)-\frac{b}{a}\sin(bT)\right)\right]}{\left[z^2-2ze^{-aT}\cos(bT)+e^{-2aT}\right]\left(a^2+b^2\right)/a}$
$\frac{z(-1)^{m-1}}{(m-1)!}\frac{\partial^{m-1}}{\partial a^{m-1}}\frac{1}{z-e^{-aT}}$ or $\frac{z(-T)^{m-1}}{(m-1)!}\left[\frac{\partial}{\partial z}z\ldots\right]^{m-1}\frac{1}{z-e^{-aT}}$	$\frac{t^{m-1}}{(m-1)!}e^{-at}, m\geq 1$	$\frac{1}{(s+a)^m}$	$\frac{1}{a^m}\left(1-(z-1)\sum_{v=1}^{m}\frac{(-a)^{v-1}}{(v-1)!}\frac{\partial^{v-1}}{\partial a^{v-1}}\frac{1}{z-e^{-aT}}\right)$ or $\frac{1}{a^m}\left(1-(z-1)\sum_{v=1}^{m}\frac{(-aT)^{v-1}}{(v-1)!}\left[\frac{\partial}{\partial z}z\ldots\right]^{v-1}\frac{1}{z-e^{-aT}}\right)$
$\frac{z(-T)^{m-1}}{(m-1)!}\left[\frac{\partial}{\partial z}z\ldots\right]^{m-1}\frac{1}{z-1}$	$\frac{t^{m-1}}{(m-1)!}, m\geq 1$	$\frac{1}{s^m}$	$(z-1)\frac{(-T)^m}{m!}\left[\frac{\partial}{\partial z}z\ldots\right]^m\frac{1}{z-1}$

T is the time-interval of equidistant sampling of signals and holding of sequences. a can be a complex number.
The given discrete-time transfer functions are based on *stair-case inputs*: $u(t)=u_i$ for $t_i\leq t<t_i+T$.

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Euler's Method

$\dot{y}=f(y,t)$

$y_{n+1}=y_n+hf(y_n,t_n)$

$t_{n+1}=h+t_n$

Runge-Kutta Fourth Order Method

$\dot{y}=f(t,y)$

$t_{n+1}=h+t_n$

$y_{n+1}=y_n+\frac{1}{6}\{k_1+2k_2+2k_3+k_4\}h$

$k_1=f(t_n,y_n)$

$k_2=f(t_n+\frac{h}{2},y_n+\frac{hk_1}{2})$

$k_3=f(t_n+\frac{h}{2},y_n+\frac{hk_2}{2})$

$k_4=f(t_n+h,y_n+hk_3)$

Feedback

Let G(z) be the z-transform OF AN OPEN LOOP SYSTEM,

Then, the closed-loop discrete transfer function is given by: $T(z)=\frac{G(z)}{1+G(z)}$