

UNIVERSITY OF KWAZULU-NATAL
School of Engineering
Discipline of Electrical, Electronic and Computer Engineering
(Howard College Campus)

Main Examinations: **June 2026**

ENEL4AA H1 : Design and Analysis of Algorithms

DURATION :2 HOURS

MARKS : 100

Examiners:

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Internal Moderator:

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Instructions:

1. Answer ALL 6 Questions.
2. Start each question on a new page, and number all your answers clearly.
3. Calculators may be used.
4. Ensure you have 4 pages, including this cover sheet.
5. **This paper may not be removed from the exam hall.**

Question 1: Analysis and Growth rate of running time of algorithms**[30 marks]**(a) State whether the following statements are **true** or **false**. Prove your answer. [3]

(i) $7n^3 + \sqrt{n} = \Omega(n^2\sqrt{n} + \sqrt{n})$

(ii) $5n^2 + \sqrt{15n^4 + 10n} + 8 = \Theta(n^2)$

(b) The running time $T(n)$ of a given algorithm verifies the following recurrence relation:

$$T(n) = 9T\left(\frac{n}{2}\right) + 7n^3 + 2n \quad (1)$$

Using the master theorem, give the solution of the recurrence relation (1) above, in the form $T(n) = O(f(n))$, where $f(n)$ is a function in terms of n . [2]

(c) Given the algorithm below.

```

int evaluate(int n)
1: eval = 0
2: for i = 1 to n do
3:   for j = 1 to i do
4:     for k = j to i+j do
5:       eval = eval+1;
6:     end for
7:   end for
8: end for
9: return eval

```

What is the growth rate of $T(n)$ of evaluate? What value does `evaluate(n)` compute?(Express your answer as function of n) [7](d) Given an array A of size n sorted in decreasing order, What is the order of running time for the binary insertion sort algorithm applied to A to sort it in increasing order. [1](e) Arrange the following functions in increasing order of their growth rate, for large n .

(i) $5^n, \log n^2, 3n, n^5, n!$ [2]

(ii) $n^3 \log n, n^4 2^n, (n+5)^8, n^3$. [1]

(f) The function f below is given.

```

function f(n)
1: if (n ≤ 1) then
2:   return 2n
3: else
4:   return 7f(n-1) - 12f(n-2)
5: end if

```

(f.1) Express in terms of n what the function f computes. [5]

(f.2) Prove by induction your answer given in (f.1). [4]

(f.3) Find the recurrence relation for the maximum running time, $T(n)$, of f and give the growth rate of $T(n)$. [5]

Question 2: Sorting Algorithms**[16 marks]**

- (a) Use merge sort to arrange the following list in increasing order:

$30, -2, 10, -4, 4, 41, 12, 3$

Show your working. What is the number of comparisons. [8]

- (b) Show that the running time of **Quicksort** is $\Theta(n^2)$ when the array A, to be sorted, contains n distinct elements and is sorted in decreasing order. [8]

Question 3: Algorithms Design Techniques**[12 marks]**

- (a) Given the undirected graph in Figure 2.

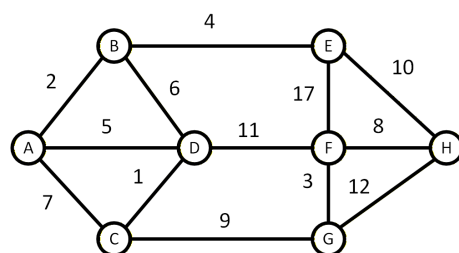


Figure 1: Undirected graph

Find the Minimum Spanning Tree for the following graph (you have to show the different steps) using Kruskal's algorithm. What is the weight of the Minimum Spanning Tree generated ? [8]

- (b) Discuss the **Divide and Conquer** paradigm. Illustrate your discussion using Binary Search. [4]

Question 4: Graph Algorithms, Trees and Ordered Sets**[18 marks]**

- (a) Given a directed Graph $G = (V, E)$ in Figure 2, use an algorithm which uses Depth-First-Search (DFS) twice to find the strongly connected components of G. Start the first DFS from A and follow the alphabetic order in the visit of adjacent vertices. [8]

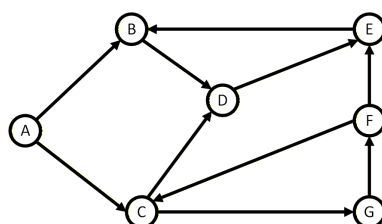


Figure 2: Directed graph

- (b) A **ternary tree** is a tree data structure in which each node has at most three child nodes. A full ternary tree has all its leaves at the same level.
- (i) Explain why such a tree has the maximum number of nodes among ternary trees of the same height. [3]
 - (ii) Derive a formula for the total number of nodes N in terms of the height h . [4]
 - (iii) If a full ternary tree has N nodes, determine its height in terms of N ? [3]

Question 5: Complexity Theory

[10 marks]

- (a) Given a non-directed graph $G = (V, E)$ with n vertices. The **NP-Chain** problem is defined as follows:
Does it exist two distinct vertices u and v in V , such that there is a cycle of length $\frac{n}{2}$ between u and v ?
Show that **NP-Chain** belongs to $\mathcal{NP}$ (Set of Non-deterministic Polynomial problems) [4]
- (b) Comment on the following statement:
"**There is no polynomial-time algorithm to solve Hamiltonian Cycle problem**" [3]
- (c) Is **Merge Sort** $\mathcal{NP}$ -complete? Justify your answer. [3]

Question 6: Algorithm Design

[20 marks]

A network of environmental sensors is deployed along a coastline to monitor ocean activity. Each sensor records a numerical reading every hour, representing wave intensity. These readings are stored in an array:

$$A[1..n]$$

Due to ocean currents and geological effects, it is suspected that certain regions exhibit sharp changes in intensity, which may indicate underwater disturbances.

A significant fluctuation is defined as a pair of indices (i, j) such that: $i < j$, and $A[i] > 2A[j]$

Such pairs are important for identifying abnormal drops in intensity over time.

- (a) Design an efficient divide and conquer algorithm to count the total number of significant fluctuations in the array. What is the running time of your algorithm? [12]
- (b) Design an algorithm that instead of counting, records all significant fluctuation pairs. What is the running time of your algorithm? [8]