University of KwaZulu-Natal

School of Engineering

Examinations: May 2026

ENEL4DCH1 : Digital Communications

Duration: **2 Hours** Marks: 100

Internal examiner: Prof H. Xu
Internal Moderator: Prof N. Pillay

External examiner: Prof Thomas Olwal

Instructions : Answer all questions.

Question 1 Waveform Coding [15 marks]

(1) [**2 marks**] The block diagram of the waveform coding is shown in Fig. 1.1

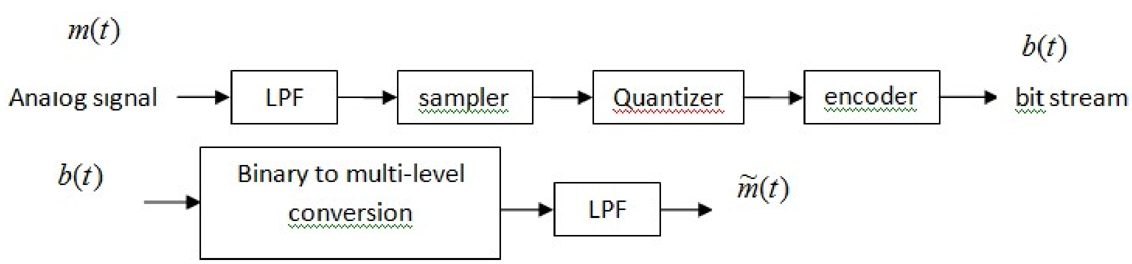

Fig. 1.1 Block Diagram of the Waveform Coding

Suppose the input analog signal $m(t)$ is a zero mean stationary process. The correlation coefficient of the input analog signal is 0.6. What waveform coding, PCM or differential PCM do you recommend?

(2) Consider an input signal $\boldsymbol{x(t)}$ with a pdf $\boldsymbol{f(x)}$ shown in Fig. 1.2 below.

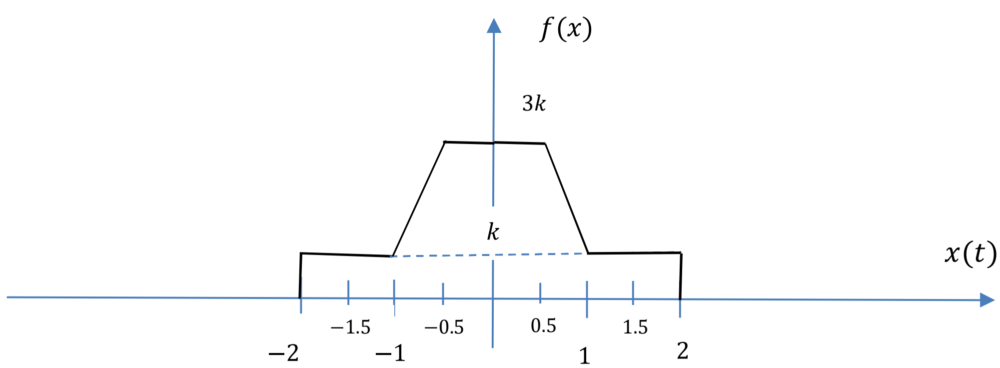

Fig. 1.2

(2.1) [**2 marks**] In Fig. 1.2, $f(x) = \begin{cases} k & -2 \le x < -1 \\ f_1(x) & -1 \le x < -0.5 \\ 3k & -0.5 \le x < 0 \\ 3k & 0 \le x < 0.5 \\ f_2(x) & 0.5 \le x < 1 \\ k & 1 \le x < 2 \end{cases}$. Both $f_1(x)$ and $f_2(x)$ are linear functions. Find k.

(2.2) [**4 marks**] Derive linear functions $f_1(x)$ and $f_2(x)$.

(2.3) [**2 marks**] The outputs of the uniform quantizer are $\{-1.5, -0.5, 0.5, 1.5\}$ for Q=4. Write an expression to calculate the variance of the quantization error for $1 \le x < 2$;

(2.4) [**5 marks**] Suppose a quantizer is employed in this sub-question. Prove the output -1.5 of the quantizer is optimal in terms of the variance of the quantization error for Q=4 and $-2 \le x < -1$.

Question 2 Information Theory: Source coding [15 marks]

A 100×100 source data has five symbols: A, B, C, D, and E. We would like to encode the source data in a lossless way.

(1) [**3 marks**] What is the minimum number of bits required if no additional symbol distribution is available?

(2) [**4 marks**] What is the minimum number of bits required if the symbols are distributed as follows: the first one indicates the symbol and the second one indicates the number of symbols corresponding to the symbol. (A,2000); (B,4000); (C,1000); (D,1000); (E,2000). Solution:

(3) [**8 marks**] For each of the above two cases, construct an optimal code assuming each symbol is coded individually.

Question 3 Information Theory: Entropy and Mutual information [15 marks]

(1) [**7 marks**] Consider an information source X emitting four symbols (x_1, x_2, x_3, x_4) with probabilities $p(x_1) = 0.4$, $p(x_2) = q_2$, $p(x_3) = q_3$ and $p(x_4) = q_4$. Prove that the source provides the maximum average information only when $p(x_2) = p(x_3) = p(x_4) = 0.2$.

(2) [**8 marks**] Let joint probability distribution of random variable A and B, $P(A, B)$ be

$$\begin{bmatrix} p(a_0,b_0) & p(a_1,b_0) & p(a_2,b_0) & p(a_3,b_0) \\ p(a_0,b_1) & p(a_1,b_1) & p(a_2,b_1) & p(a_3,b_1) \\ p(a_0,b_2) & p(a_1,b_2) & p(a_2,b_2) & p(a_3,b_2) \\ p(a_0,b_3) & p(a_1,b_3) & p(a_2,b_3) & p(a_3,b_3) \end{bmatrix} = \begin{bmatrix} 1/8 & 1/16 & 1/32 & 1/32 \\ 1/16 & 1/8 & 1/32 & 1/32 \\ 1/16 & 1/16 & 1/16 & 1/16 \\ 1/4 & 0 & 0 & 0 \end{bmatrix}$$

Find $H(B|a_1)$.

Question 4 Channel coding: convolutional coding [15 marks]

(1) [**8 marks**] Consider a rate 1/2 linear convolutional code with K=3. It is assumed that the initial state of the convolutional encoder is "00". It is also assumed that the final state is also "00". Given an input sequence "1000", where leftmost bit is the first input bit, the output sequence is "1 1 0 1 1 1 0 0" (where leftmost two bits is the first output bits). Find the output sequence for the input sequence "101100".

(2) Consider another convolutional code with rate 1/3 and K=3 is shown in Fig. 4.1 below.

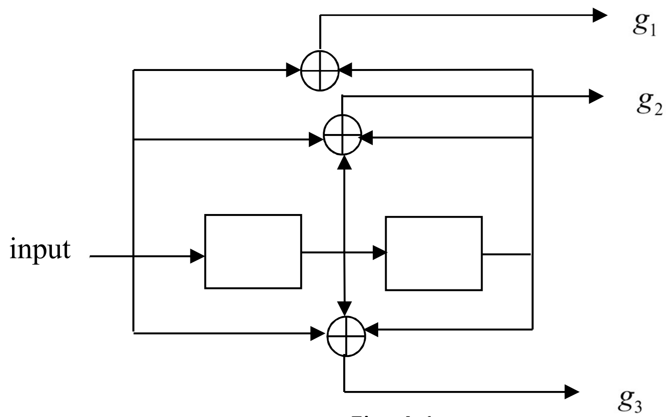

Fig. 4.1

(2.1) [**2 marks**] Show the convolutional code generator in both octal and polynomial;

(2.2) [**2 marks**] Draw the state transition table;

(2.3) [**3 marks**] Find the minimum Hamming distance (free Hamming distance) of the code.

Question 5 Channel Coding: linear block coding [15 marks]

(1) Consider a (6,3) linear block code C with generator matrix

$$G = [P_{3\times3}\ I_3]$$

where $P_{3\times3}$ is parity check matrix.

The codeword of C is expressed as $C = [c_1\ c_2\ c_3\ c_4\ c_5\ c_6]$. The pictorial representation of the parity check matrix H is shown in Fig. 5.1

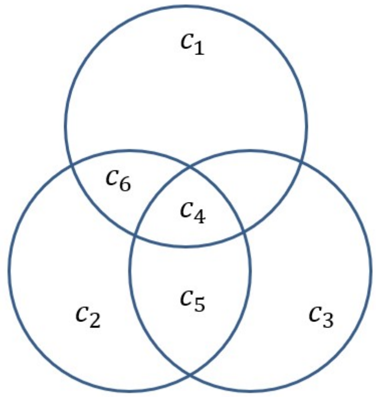

Fig. 5.1 pictorial representation of the parity check matrix H

(1.1) [**3 marks**] Determine the parity check matrix H of the code C from Fig. 5.1;

(1.2) [**2 marks**] Determine the generator matrix G of the code C;

(2) We have implemented (7,4) Hamming code over additive white Gaussian noise (AWGN) in Prac 1. The whole steps of implementation are shown in Table 5.1.

(2.1) [**5 marks**] Write Matlab code to implement generator G, and standard array S. Note that standard array S is used for finding an error pattern based on syndrome $\boldsymbol{s}$, not for finding an error position**.** Use $\boldsymbol{s} = [1\ 0\ 1]$ as an example to explain how to find the error pattern based on your matlab standard array S.

(2.2) [**1 marks**] Write Matlab code to generate $b_3 b_2 b_1 b_0$;

(2.3) [**2 marks**] Write Matlab code to convert $[c_1 c_2 c_3 c_4 c_5 c_6 c_7]$ into $[x_1 x_2 x_3 x_4 x_5 x_6 x_7]$ based on $\begin{cases} x_i = +1, & c_i = 0 \\ x_i = -1, & c_i = 1 \end{cases}$;

(2.4) [**2 marks**] Write Matlab code to convert SNR_dB into decimal;

Table 5.1: Steps for implementing (7,4) Hamming code

Theory	Matlab code
Initialization: SNR_dB =[0:2:8] dB; Generator: $G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$ Parity check matrix: $H = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ Standard array $\boldsymbol{S}$:	

<table>
<tr><td>
<table>
<tr><th>syndrome</th><th>Error pattern</th></tr>
<tr><td>0 0 0</td><td>0 0 0 0 0 0 0</td></tr>
<tr><td>0 0 1</td><td>0 0 0 0 0 0 1</td></tr>
<tr><td>0 1 0</td><td>0 0 0 0 0 1 0</td></tr>
<tr><td>1 0 0</td><td>0 0 0 0 1 0 0</td></tr>
<tr><td>1 1 1</td><td>0 0 0 1 0 0 0</td></tr>
<tr><td>1 1 0</td><td>0 0 1 0 0 0 0</td></tr>
<tr><td>1 0 1</td><td>0 1 0 0 0 0 0</td></tr>
<tr><td>0 1 1</td><td>1 0 0 0 0 0 0</td></tr>
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Encoding
Generating information bits: $b_3b_2b_1b_0$;
Encoding $C = [b_3b_2b_1b_0]G$;
$C = [c_1c_2c_3c_4c_5c_6c_7]$;
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<tr><td>
Transmission
Converting c_i into x_i, $\begin{cases} x_i = +1, & c_i = 0 \\ x_i = -1, & c_i = 1 \end{cases}$
Transmission: $y_i = x_i + n_i$, $n_i \sim N(0, \sigma^2)$
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Detection:
Received codeword: $\begin{cases} r_i = 0, & y_i > 0 \\ r_i = 1, & y_i < 0 \end{cases}$
Calculate syndrome:
$$\boldsymbol{s} = [r_1r_2r_3r_4r_5r_6r_7]H^T$$
Based on S Find error pattern e;
Estimated transmitted codeword: $\hat{C} = r + e$
</td><td></td></tr>
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Question 6 Digital modulation [25 marks]

(1) [**2 marks**] Sketch double-sided spectral density of an AWGN distributed as $N(0,1)$.

(2) [**2 marks**] what the difference between bit error rate and bit error probability?

(3) [**2 marks**] Explain why the error performance of differential PSK is worse than PSK.

(4) [**3 marks**] An encoded transmission system which has been implemented in Prac 1 is shown in Fig. 6.1 below.

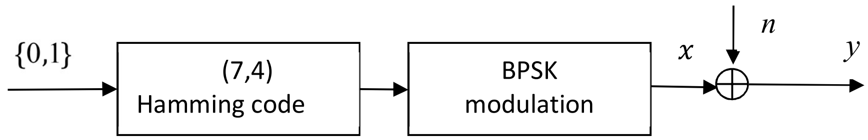

Fig. 6.1

The binary symbols 0 and 1 with equal probability are encoded, and then are modulated. At receiver $y = \sqrt{\rho}x + n$, where ρ is the transmitted power, x is the BPSK signal ($x \in \{-1, +1\}$), and n is a white Gaussian noise with distribution $f(n) = \frac{1}{\sqrt{2\pi}} e^{-\frac{n^2}{2}}$. Suppose that the error probability of demodulation is p_b. Show the bit error probability $p_e(\rho)$ after decoding.

Note: The signal-to-noise ratio (SNR) value refers to the ratio between symbol and noise power.

(5) [**5 marks**] In limited bandwidth channel, the channel capacity C is given by

$$C = B \times log_2\left(1 + \frac{S}{N_0 B}\right)$$

where B is the channel bandwidth and $\frac{S}{N_0 B}$ is the signal-to-noise ratio at the receiver.

Explain why C does not always increase if B increases.

(6) A 4PAM signal is transmitted over an AWGN channel. The received signal is given by

$$y = x + n\,,$$

where $x \in [-3, -1, 1\ 3]$ is the 4PAM signal, $P(x = -3) = P(x = -1) = P(x = 1) = P(x = 3) = \frac{1}{4}$, n is the AWGN with $n \sim N\left(0, \frac{\varepsilon}{E_s}\right)$ and $\varepsilon = E[x^2]$.

(6.1) [**8 marks**] Derive $p(\hat{x} = 1|x = 1)$ in terms of Gaussian Q-function $Q(x)$, where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{-\infty} e^{-\frac{t^2}{2}} dt$.

(6.2) [**3 marks**] Derive $p(\hat{x} = -1|x = -1)$ in terms of Gaussian Q-function $Q(x)$, where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{-\infty} e^{-\frac{t^2}{2}} dt$.

Note that no mark is awarded if you directly write the answer.