School of Engineering, Electrical, Electronic and Computer Engineering

Course code and name: ENEL4DS, Digital Signal Processing
Main Examination - 2026

Total marks: 60
Duration: 2 hours

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Instructions to candidates:

- This paper consists of **4 pages and 3 questions**.
- All questions must be answered in full for full marks to be awarded (**All working must be shown**).
- All relevant assumptions must be made and documented in your workings.
- Programmable calculators may be used provided all memory is cleared.
- No notes of any kind are allowed. Useful data is attached at the back of the question paper.
- Organize your solutions neatly. Use diagrams and equations to aid your explanation where possible.

Question 1 [20 marks]

1.1 A sequence is given by $x[n] = \{4 + j \quad -2 - j \quad 4 \quad -5 - 2j\}$ for $-1 \leq n \leq 2$. **Determine** the conjugate-symmetric and conjugate anti-symmetric parts of the sequence. [4]

1.2 Consider the sequence at the output of a system is given as:

$$y[n] = n\left(\frac{4}{3}\right)^n \sum_{k=-\infty}^{0} \delta[n-k]$$

Determine the energy of the sequence. [7]

1.3 Consider the sequence:

$$x[n] = 3\sin\left(\frac{n\pi}{2}\right)\{u[n+3] - u[n-4]\}$$

Sketch the sequences:

A. $x[-n-3]$ [3]

B. $x[n^2-n+1]$ [3]

1.4 Suppose that $x_b[n]$ is the conjugate symmetric part of a sequence $x[n]$. **Determine** the symmetry of the real and imaginary parts of $x_b[n]$. [3]

Question 2 [20 marks]

2.1 An N-point circular convolution between two sequences can be attained from the linear convolution of the same two sequences. Consider the sequences:

$$x[n] = [1\ 3\ 1\ 4], n \in [1:4]$$
$$h[n] = [2\ 4\ 5], n \in [1:3]$$

A. **Determine** the 4-point circular convolution by extending the sequences into periodic sequences. [5]

B. **Write** an expression relating circular convolution to linear convolution. [1]

C. **Show** using calculation how the linear convolution of the sequences above are used to determine the 4-point circular convolution attained in 2.1.A. [6]

2.2 Consider the complex-valued sequence $x[n]$, described as follows:

$$x[n] = \{x[0] \quad x[3] \quad x[5] \quad x[7] \quad x[9] \quad x[11] \quad x[13] \quad x[15]\}$$

A. A fast approach that may be employed to solve the DFT is the decimation-in-time FFT. **Use** a block diagram to represent the decimation-in-time computation using only 2-point DFTs when $N = 8$. **Label** the diagram fully. [5]

B. **Determine** the percentage reduction in computational complexity compared to the DFT approach. [3]

Question 3 [20 marks]

3.1 **Determine** the z-transform of the following sequence:

$$x[n] = n\left(\frac{1}{6}\right)^n u[-n-3]$$ [6]

3.2 Consider the sequence $x[n] = \{2\ -2\ \ 5\ \ 2\}, n \in [-1{:}\,2]$. **Compute** the integral $I_1 = 4\pi \int_{-\pi}^{\pi} X\left(e^{jw}\right)e^{-jw}\, dw$ without computing the DTFT of the sequence. [3]

3.3 The transfer function for a particular digital filter is given as:

$$H(z) = \frac{(0.55 + 0.35z^{-1} + 0.45z^{-2})}{(1 + 0.55z^{-1} + 0.65z^{-2})} \frac{z^{-1}}{(1 - 0.65z^{-1})}$$

A. **Generate** the Direct Form II realization structure. [3]
B. **Generate** the Cascade Form realization structure. [3]

3.4 Consider the unit sample response:

$$h[n] = u[n] - u[n-1] - \beta(u[n-1] - u[n-2])$$

Find the magnitude and phase responses of the sequence $h[n]$, where β is a real-valued constant. [5]

DATASHEET

Closed-form expressions
$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$
$\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$
$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \lvert a \rvert < 1$
$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2}, \ \lvert a \rvert < 1$

Table of DTFT Pairs	
$\delta[n]$	1
$\delta[n-n_o]$	$e^{-jn_o\omega}$
1	$2\pi\delta[\omega]$
$e^{jn\omega_o}$	$2\pi\delta[\omega-\omega_o]$
$\alpha^n u[n], \lvert\alpha\rvert < 1$	$\frac{1}{1-\alpha e^{-jw}}$
$-\alpha^n u[-n-1], \lvert\alpha\rvert > 1$	$\frac{1}{1-\alpha e^{-jw}}$
$(n+1)\alpha^n u[n], \lvert\alpha\rvert < 1$	$\frac{1}{(1-\alpha e^{-jw})^2}$
$\cos n\omega_o$	$\pi\delta[\omega+\omega_o] + \pi\delta[\omega-\omega_o]$

Table of ZT Pairs		Region of convergence
$\delta[n]$	1	$\forall\, z$
$\alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}$	$\|z\| > \|\alpha\|$
$-\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$\|z\| < \|\alpha\|$
$n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\|z\| > \|\alpha\|$
$-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\|z\| < \|\alpha\|$
$\cos n\omega_o\, u[n]$	$\frac{1-(\cos\omega_o)z^{-1}}{1-2(\cos\omega_o)z^{-1}+z^{-2}}$	$\|z\| > 1$
$\sin n\omega_o\, u[n]$	$\frac{(\sin\omega_o)z^{-1}}{1-2(\cos\omega_o)z^{-1}+z^{-2}}$	$\|z\| > 1$

$d = \left[\frac{\left(1-\delta_p\right)^{-2}-1}{\delta_s^{-2}-1}\right]^{1/2}$	$q = q_o + 2q_o^5 + 15q_o^9 + 150q_o^{13}$	$s = \frac{2}{T_s}\frac{1-z^{-1}}{1+z^{-1}}$
$N \geq \frac{\log\left(\frac{16}{d^2}\right)}{\log\left(\frac{1}{q}\right)}$	$N = \frac{-10\log\delta_p\delta_s - 13}{14.6\Delta f}$	$\omega = 2\tan^{-1}\left(\frac{\Omega T_s}{2}\right)$
$q_o = \frac{1}{2}\left[\frac{1-(1-k^2)^{1/4}}{1+(1-k^2)^{1/4}}\right]$	$\|H_a(j\Omega)\|^2 = \frac{1}{1+\left(\frac{j\Omega}{j\Omega_c}\right)^{2N}}$	$\epsilon = \left(\frac{\Omega_p}{\Omega_c}\right)^N$
$T_N(x) = \begin{cases} \cos(N cos^{-1}x) & \|x\| \leq 1 \\ \cosh(N cosh^{-1}x) & \|x\| > 1 \end{cases}$	$\|H_a(j\Omega)\|^2 = \frac{1}{1+\epsilon^2 T_N^2\left(\Omega/\Omega_p\right)}$	$k = \frac{\Omega_p}{\Omega_s}$