School of Engineering, Electrical, Electronic and Computer Engineering

Course code and name: ENEL3CO, Communications
2025 Main Examination

Marks: 100
Duration: 3 hours

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Instructions to candidates:

- This paper consists of **TWO Sections with TWO questions each. Answer each SECTION in a NEW answer book.**
- All questions must be answered in full for full marks to be awarded (**All working must be shown**).
- All relevant assumptions must be made and documented in your workings. Organize your solutions neatly.
- Programmable calculators may be used provided all memory is cleared.
- No notes of any kind are allowed. Useful data is attached at the back of the question paper.

SECTION A
Question 1 [25 marks]

1.1 A linear time-invariant channel has the following amplitude characteristics

$$H(f) = \frac{1}{2}\prod\left(\frac{f}{400}\right) + \frac{1}{2}\prod\left(\frac{f}{1000}\right)$$

The phase characteristics of the channel is $-\frac{\pi}{2}$ for $f \geq 0$, and $\frac{\pi}{2}$ for $f < 0$.
The signal

$$x(t) = sin(200\pi t) + cos(200\pi t) + sin(600\pi t) + cos(4000\pi t)$$

is applied to the channel.

A. **Plot** the input power spectral density. [5]
B. **Plot** the output power spectral density. [5]
C. **Compute** the signal $y(t)$ at the output of the channel. [5]

1.2 **Compute** the convolution of the signals $x(t) = \prod(t)$ and $h(t) = \prod(t-2)$. [5]

1.3 **Compute** the Fourier transform of the signal below based on first principles. [5]

$$x(t) = e^{-\alpha|t|}\,, \alpha > 0$$

Question 2 [25 marks]

2.1 The probability density function of a random variable X is shown below.

$$x(t) = \begin{cases} A(x+2), & -2 \le x \le -1 \\ A, & -1 \le x \le 1 \\ -A(x-2), & 1 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

A. **Compute** the cumulative distribution function. [6]
B. **Compute** the variance of the random variable. [4]

2.2 A random process is expressed as
$Y(t) = sin(2\pi f_0 t + A) + cos(2\pi f_0 t + B) + C$ where A, B, and C are independent random variables uniformly distributed in $[0, 2\pi)$.
Assume $f_0 = 100\ Hz$.

A. **Provide** three expressions for realisations of the random process. [3]
B. **Compute** the mean of the random process. [3]

2.3 A low-noise receiver for a satellite earth station consists of an antenna and two radio frequency (RF) subsystems cascaded together. The parameters are given as follows. [6]

RF subsystem 1 – Gain 10 dB and noise figure 3 dB
RF subsystem 2 – Gain 16 dB and noise figure 2 dB

Assume the bandwidth of both subsystems is 25 MHz and the effective temperature of the antenna is 60 K. **Determine** the effective noise temperature of the receiver system.

2.4 **Provide** a brief description of ergodic processes. [3]

SECTION B
Question 1 [25 marks]

1.1 Consider the following transmit signal:

$$x(t) = 120[1 + 0.2\cos(2\pi 10^3 t) + \cos(2\pi 2.10^3 t)]\cos(2\pi 10^6 t)$$

A. **What** type of modulation has been used to yield the above signal? **Motivate** your answer. [2]
B. **Show** using math how a coherent demodulator may be used to recover the message signal at the receiver. [4]
C. **Suggest** a method that may be used to reduce the complexity of the receiver apparatus. **Explain** a possible disadvantage of your suggested method. [2]
D. The upper sidebands are removed from the signal $x(t)$. **Sketch** the double-sided amplitude-frequency spectrum of the resulting signal. [3]
E. The power of the sidebands in 1.1.D is found to be approximately 25 W for a connected load impedance. **Determine** the value of the load impedance. [2]

1.2 **Validate** the method of natural sampling of a message signal $m(t)$. **Provide** the mathematical descriptions of the time domain and frequency domain signals. **Include** the hypothetical sketches of the signals in both the time and frequency domains. [6]

1.3 Consider the signal $x(t) = A\cos[w_c t + \beta \sin w_m t]$. The carrier amplitude was found as 120 V. The maximum deviation frequency was found as 20 kHz, while the bandwidth of $x(t)$ was found as 60 kHz from Carson's bandwidth rule.

Draw and fully label the single-sided power-frequency spectrum for this modulated signal. Assume a load impedance of 75 Ohm. [6]

Question 2 [25 marks]

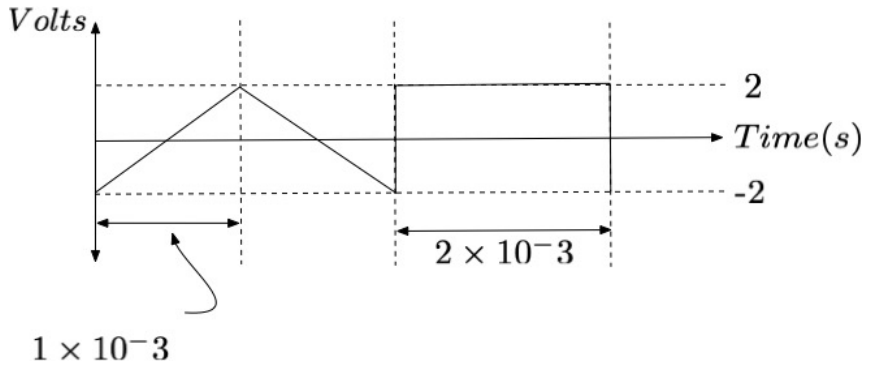
2.1 A DSB-AM transmitter is modulated by a video testing signal $v(t) = 0.6 + 0.4\sin\omega_1 t$. The carrier has a peak amplitude of 150 Volts and is sinusoidal with an amplitude of 100% at $t = 0$. The number of cycles of the carrier frequency in each cycle of the message is found as 20 times larger than the modulation index.

Sketch the modulated signal in the time domain. Draw at least two cycles of the message envelope. [6]

2.2 Consider a signal $m(t) = \cos w_0 t$, where $w_0 = 2\pi f_0$. **Illustrate** the effect of

under-sampling of $m(t)$ for a sampling rate of $f_s = \frac{3}{2} f_0$. Use the time domain plot to depict the samples. [4]

2.3 Consider the following message signal:



Sketch the FM and PM transmit signals. Assume the frequency and phase deviation constants are given as $k_f = 2\pi 10^3$ and $k_p = 10\pi$. The carrier frequency is 108 MHz and the carrier amplitude is 100 V.
Note that you should show all relevant mathematical expressions. [9]

2.4 The Indirect method of Armstrong employs frequency multiplication to convert a narrowband angle modulated signal into a wideband angle modulated signal.

Using the indirect method of Armstong, **design** for the following PM output:

$$f_c = 89\ MHz$$
$$\Delta f = 55\ kHz$$

Assume that the narrowband signal is centered at a frequency of 200 kHz with a modulation index of 0.1.
Present your design with all the important values included. Make use of only doublers and triplers. An oscillator with an adjustable range of 16 MHz to 17 MHz is available. [6]

DATASHEET

$\text{Propagation loss factor} = \left(\frac{\lambda}{4\pi d}\right)^2$ where λ is the transmission wavelength and d is the distance of the satellite from the earth station.

$G_R = \frac{4\pi}{\lambda^2} A_R$, where A_R is the effective antenna aperture area.

Boltzmann's constant $\mathrm{k} = 1.38\text{x}10^{-23}\text{J/K}$

Standard temperature T_o = 290 K

Speed of light = $3\text{x}10^8$ m/s

TABLE OF FOURIER TRANSFORMS

Time Domain	Frequency Domain
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$e^{-j2\pi f t_0}$
$e^{j2\pi f_0 t}$	$\delta(f - f_0)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$
$\sin(2\pi f_0 t)$	$-\frac{1}{2j}\delta(f + f_0) + \frac{1}{2j}\delta(f - f_0)$
$\Pi(t)$	$\text{sinc}(f)$
$\text{sinc}(t)$	$\Pi(f)$
$\Lambda(t)$	$\text{sinc}^2(f)$
$\text{sinc}^2(t)$	$\Lambda(f)$
$e^{-\alpha t}u_{-1}(t), \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$
$te^{-\alpha t}u_{-1}(t), \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$
$e^{-\alpha\lvert t\rvert}$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$
$e^{-\pi t^2}$	$e^{-\pi f^2}$
$\text{sgn}(t)$	$1/(j\pi f)$
$u_{-1}(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\delta'(t)$	$j2\pi f$
$\delta^{(n)}(t)$	$(j2\pi f)^n$
$\frac{1}{t}$	$-j\pi\,\text{sgn}(f)$
$\sum_{n=-\infty}^{n=+\infty}\delta(t - nT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{n=+\infty}\delta(f - \frac{n}{T_0})$

β: / n	0.5	1	2	3	4	5	6	7	8	9	10
0	0.9385	0.7652	0.2239	−0.2601	−0.3971	−0.1776	0.1506	0.3001	0.1717	−0.09033	−0.2459
1	0.2423	0.4401	0.5767	0.3391	−0.06604	−0.3276	−0.2767	−0.004683	0.2346	0.2453	0.04347
2	0.03060	0.1149	0.3528	0.4861	0.3641	0.04657	−0.2429	−0.3014	−0.1130	0.1448	0.2546
3	0.002564	0.01956	0.1289	0.3091	0.4302	0.3648	0.1148	−0.1676	−0.2911	−0.1809	0.05838
4		0.002477	0.03400	0.1320	0.2811	0.3912	0.3576	0.1578	−0.1054	−0.2655	−0.2196
5			0.007040	0.04303	0.1321	0.2611	0.3621	0.3479	0.1858	−0.05504	−0.2341
6			0.001202	0.01139	0.04909	0.1310	0.2458	0.3392	0.3376	0.2043	−0.01446
7				0.002547	0.01518	0.05338	0.1296	0.2336	0.3206	0.3275	0.2167
8					0.004029	0.01841	0.05653	0.1280	0.2235	0.3051	0.3179
9						0.005520	0.02117	0.05892	0.1263	0.2149	0.2919
10						0.001468	0.006964	0.02354	0.06077	0.1247	0.2075
11							0.002048	0.008335	0.02560	0.06222	0.1231
12								0.002656	0.009624	0.02739	0.06337
13									0.003275	0.01083	0.02897
14									0.001019	0.003895	0.01196
15										0.001286	0.004508
16											0.001567

Table of Bessel values

Bessel Plots

Table of Useful Trignometric formulae

$\sin a\cos b = 0.5\left[\sin(a+b)+\sin(a-b)\right.$
$\cos a\sin b = 0.5\left[\sin(a+b)-\sin(a-b)\right.$
$\cos a\cos b = 0.5\left[\cos(a+b)+\cos(a-b\right.$
$\sin a\sin b = 0.5\left[\cos(a-b)-\cos(a+b\right.$
$\sin a+\sin b = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$
$\cos a+\cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right.$
$\sin(a\pm b) = \sin a\cos b \pm \cos a\sin b$
$\cos(a\pm b) = \cos a\cos b \mp \sin a\sin b$