University of KwaZulu-Natal

ENEL 3CS H2 Control Systems I

MAIN EXAMINATION

Duration: 2 hours. Maximum mark: 100. Date: 3rd November 2025

Examiner: Prof. J. T. Agee Moderator: Dr. R. Seare

Instructions: *This question paper has four question, spread over three pages. Answer all four questions. A two-page formula sheet is attached at the end of the paper.*

You may write your answers in blue or black ink. Do not write in pencil or any colour of ink other than the two colours advised, otherwise such work would not be marked.

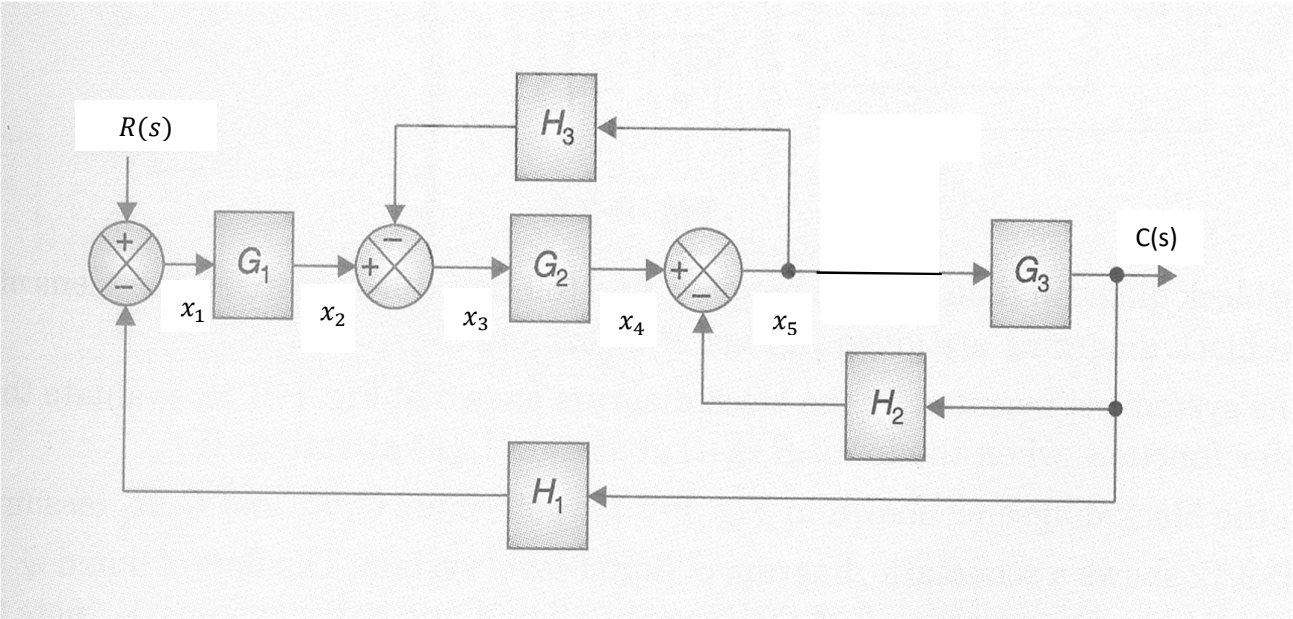
Plots not using the appropriate graph sheets will not be marked.

No programmable calculators allowed during this examination.

Included: Graph paper.

Question One [28 Marks]

Consider the block diagram shown in Figure 1. The variables in the block diagram are: the input R(s), x_1, x_2, x_3, x_4, x_5 and the output C(s).



1.1 Write the equations for the system variables x_1, x_2, x_3, x_4, x_5, and C(s). [6]

1.2 Using block-diagram algebra, derive the expression for $C(s)$, where $C(s)$ is the output of the block diagram due the input $R(s)$. [16]

1.3 Draw the equivalent signal flow graph for the system given in Figure 1. [6]

Question Two [24 marks]

A third-order, open-loop control system has the transfer function given by

$$G_2(s) = \frac{K}{s(s+7)(s+11)}:$$

2.1 Derive the characteristic equation of the given system. [2]

2.2 Perform Routh-Hurwitz stability analysis of $G_2(s)$ and determine the condition(s) required to make the given system stable. [8]

2.3 Evaluate K_{Cr} , ω_{Cr}, P_{Cr} , respectively the critical gain, critical frequency and the critical period of oscillations of $G_2(s)$. [6]

2.4 Design the PID controller for the system. [8]

Question Three [19 marks]

3.1 A fourth-order, open loop transfer function is given by $G_3(s) = \frac{50}{s^2(s+2)(s+8)}$. Evaluate the following time-based indices for the transfer function.

(a) The positional error e_p. [2]

(b) The velocity error e_v. [2]

(c) The acceleration error e_a. [3]

3.2 Evaluate the following indices of performance for the second-order, closed-loop control system given by $T(s) = \frac{50}{s^2+5s+36}$:

(i) The settling time t_s. [5]

(ii) The peak time t_p. [2]

(iii) The rise time t_r. [3]

(iv) The percentage overshoot %OS. [2]

Question Four [29 marks]

Using the graph paper provided, make to scale, a detailed root-locus plot of the control system given by the transfer function $G_4(s) = \frac{K}{s(s+5)(s+8)}$. Show all asymptotes, asymptote angles, the centroid, break-away point and any crossing of the imaginary axis on the plot. Hence, evaluate the system's gain K^*, and the steady state error if the controlled system were to have a damping factor of $\xi = 0.5135$.

Modeling

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} \quad (2.97); \qquad \frac{V_o(s)}{V_i(s)} = \frac{Z_1(s) + Z_2(s)}{Z_1(s)} \quad (2.104)$$

$$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2} \quad (2.133); \qquad \frac{T_2}{T_1} = \frac{\theta_1}{\theta_2} = \frac{N_2}{N_1} \quad (2.135)$$

$$\left(\frac{\text{Number of teeth of gear on } \textit{destination} \text{ shaft}}{\text{Number of teeth of gear on } \textit{source} \text{ shaft}} \right)^2 \quad \text{(see after 2.138)}$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]} \quad (2.153)$$

$$\frac{K_t}{R_a} = \frac{T_{\text{stall}}}{e_a} \quad (2.162); \quad K_b = \frac{e_a}{\omega_{\text{no-load}}} \quad (2.163)$$

$$T(s) = \frac{Y(s)}{U(s)} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \quad (3.73)$$

Time Response

$$T_r = \frac{2.2}{a} \quad (4.9); \quad T_s = \frac{4}{a} \quad (4.10)$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (4.22)$$

$$\%OS = e^{-(\pi\zeta/\sqrt{1-\zeta^2})} \times 100 \quad (4.38)$$

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} \quad (4.39)$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \quad (4.34); \quad T_s = \frac{4}{\zeta\omega_n} \quad (4.42)$$

Steady-State Error

$$e(\infty) = e_{\text{step}}(\infty) = \frac{1}{1 + \lim_{s\to 0} G(s)} \quad (7.30); \quad K_p = \lim_{s\to 0} G(s) \quad (7.33)$$

$$e(\infty) = e_{\text{ramp}}(\infty) = \frac{1}{\lim_{s\to 0} sG(s)} \quad (7.31); \quad K_v = \lim_{s\to 0} sG(s) \quad (7.34)$$

$$e(\infty) = e_{\text{parabola}}(\infty) = \frac{1}{\lim_{s\to 0} s^2 G(s)} \quad (7.32); \quad K_a = \lim_{s\to 0} s^2 G(s) \quad (7.35)$$

Root Locus

$$\angle KG(s)H(s) = -1 = 1\angle(2k+1)180° \quad (8.13)$$

$$\sigma_a = \frac{\sum \text{finite poles} - \sum \text{finite zeros}}{\#\text{ finite poles} - \#\text{ finite zeros}} \quad (8.27)$$

$$\theta_a = \frac{(2k+1)\pi}{\#\text{ finite poles} - \#\text{ finite zeros}} \quad (8.28)$$

$$\theta = \sum \text{finite zero angles} - \sum \text{finite pole angles}$$

$$K = \frac{1}{|G(s)H(s)|} = \frac{1}{M} = \frac{\prod \text{finite pole lengths}}{\prod \text{finite zero lengths}} \quad (8.51)$$

Frequency Response

$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \quad (10.52); \quad \omega_p = \omega_n\sqrt{1-2\zeta^2} \quad (10.53)$$

$$\omega_{BW} = \omega_n\sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} \quad (10.54)$$

$$\Phi_M = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}} \quad (10.73)$$

$$\phi_{\max} = \tan^{-1}\frac{1-\beta}{2\sqrt{\beta}} = \sin^{-1}\frac{1-\beta}{1+\beta} \quad (11.11)$$

$$\omega_{\max} = \frac{1}{T\sqrt{\beta}} \quad (11.9); \quad |G_c(j\omega_{\max})| = \frac{1}{\sqrt{\beta}} \quad (11.12)$$

State Space

$$\mathbf{C_M} = [\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A^2B} \quad \cdots \quad \mathbf{A^{n-1}B}] \quad (12.26)$$

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{BK})\mathbf{x} + \mathbf{B}r; \quad y = \mathbf{Cx} \quad (12.3); \quad \mathbf{O_M} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{n-1} \end{bmatrix} \quad (12.79)$$

$$\dot{\mathbf{e}}_{\mathbf{x}} = (\mathbf{A} - \mathbf{LC})\mathbf{e}_{\mathbf{x}}; \quad y - \hat{y} = \mathbf{Ce}_{\mathbf{x}} \quad (12.64)$$

Digital Control

$$e^*(\infty) = \lim_{z\to 1}(1 - z^{-1})E(z) \quad (13.66)$$

$$K_p = \lim_{z\to 1} G(z) \quad (13.70); \quad K_v = \frac{1}{T}\lim_{z\to 1}(z-1)G(z) \quad (13.73)$$

$$K_a = \frac{1}{T^2}\lim_{z\to 1}(z-1)^2 G(z) \quad (13.75)$$

$$T_r = \frac{1}{\omega_n}\left\{1.76\xi^3 - 0.417\xi^2 + 1.039\varsigma + 1\right\}$$

$$s_1, s_2 = -\xi\omega_n + \omega_n\sqrt{1-\xi^2}$$

Lag control: $G_C(s) = K_C\left(\frac{s+1/T}{s+1/\alpha T}\right); \alpha > 1, K_C = \frac{1/\alpha T}{1/T} = \frac{1}{\alpha}$

Lead Control:

$$G_C(s) = \frac{1}{\beta}\left(\frac{s+1/T}{s+1/\beta T}\right); \beta < 1; \qquad G_C(j\omega_{\max}) = \frac{1}{\sqrt{\beta}}$$

$$\omega_{\max} = \frac{1}{T\sqrt{\beta}}; \phi_{\max} = \sin^{-1}\left\{\frac{1-\beta}{1+\beta}\right\}, \beta = \frac{1-\sin\phi_{\max}}{1+\sin\phi_{\max}}$$

	K_c	T_i	T_d
P	K_0		
PI	$0.9K_0$	$3.3\tau_{dead}$	
PID	$1.2K_0$	$2\tau_{dead}$	$0.5\tau_{dead}$

K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$