UNIVERSITY OF KWAZULU-NATAL

School of Engineering
Electrical, Electronic & Computer Engineering

MAIN EXAMINATION 2025

ENEL4TB SELECTED TOPICS IN ELECTRONIC ENGINEERING 2 – MICROWAVE ENGINEERING

Time allowed: 2 hours **Maximum marks:** 50

Instructions to Candidates:

1. This paper contains 4 questions.
2. Answer **ALL FOUR** questions.
3. This paper has 12 pages excluding the cover page.
4. All questions carry marks as indicated against them.
5. Programmable calculators may not be used in this examination.
6. Students may bring one A4 page of formulae sheet into the examination room.
7. The notations and symbols have their usual meaning.

The following materials are provided:

1. Vector operators
2. Formulae sheet

Examiner

P Kumar

Moderators

T Walingo
A Alonge

Question 1 [13 Marks]

(a) A rectangular brass ($\sigma_c = 1.37 \times 10^7\ S/m$) waveguide with dimensions $a = 1.07\ cm$ and $b = 0.43\ cm$ operates in the dominant mode at frequency 11 GHz. If the waveguide is filled with Teflon ($\mu_r = 1, \varepsilon_r = 2.08, tan\delta = 0.0004$), determine:

(i) The cutoff frequency of the dominant mode. [2]

(ii) The attenuation constant α_d (in Np/m). [4]

(iii) The attenuation constant α_c (in Np/m). [4]

(b) Can an air-filled circular metallic waveguide with $a = 0.3\ cm$ be used at 20 GHz for the single mode propagation? Justify your answer. [3]

Question 2 [12 Marks]

(a) Design a single-section quarter-wave transformer to match a 200 Ω load to a 50 Ω line. Determine the percent bandwidth of this transformer for SWR≤ 2. [4]

(b) A four-section Binomial matching transformer is to be designed to match a 15 Ω load to a 50 Ω line at a center frequency of 1 GHz. Determine:

(i) the characteristic impedances. [6]

(ii) the bandwidth (in %) for $\Gamma_m = 0.05$. [2]

Question 3 [13 Marks]

(a) Design a four-section coupled line bandpass filter with a 0.5 dB equal ripple response. The center frequency, the bandwidth and the impedance are 2.45 GHz, 10% and 50 Ω, respectively. Determine the required even-mode and odd-mode impedances of the coupled line sections and calculate the expected attenuation at 2.1 GHz. [8]

(b) Transmitting and receiving antennas operating at 1 GHz with gains of 20 dB and 15 dB, respectively, are separated by a distance of 1 km. Find the maximum power delivered to the load when the input power to the transmitting antenna is 150 W. Assume that the antennas are polarization-matched. [5]

Question 4 [12 Marks]

(a) The radiation intensity of an antenna is given by:

$$U(\theta,\phi) = \begin{cases} \frac{0.342}{sin(\theta)} & 0 \le \theta \le \frac{\pi}{3} \\ 0 & elsewhere \end{cases} \quad for\ 0 \le \emptyset \le 2\pi$$

Determine the expression for the gain of the antenna. [4]

(b) Determine the width for a 60 Ω copper stripline conductor with $b = 0.5\ cm$ and $\varepsilon_r = 2.2$. If the dielectric loss tangent is 0.001 and the operating frequency is 10 GHz, calculate the total attenuation in Np/m. Assume the conductor thickness of $t = 0.001\ cm$ and the surface resistance of copper at 10 GHz of $R_s = 0.026\ \Omega$. [8]

APPENDIX 1: VECTOR OPERATORS AND SOME PHYSICAL CONSTANTS

Coordinate Transformations

Rectangular to Cylindrical:

$$\begin{bmatrix} A_\rho \\ A_\emptyset \\ A_z \end{bmatrix} = \begin{bmatrix} cos\emptyset & sin\emptyset & 0 \\ -sin\emptyset & cos\emptyset & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Cylindrical to Rectangular:

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} cos\emptyset & -sin\emptyset & 0 \\ sin\emptyset & cos\emptyset & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\emptyset \\ A_z \end{bmatrix}$$

Rectangular to Spherical:

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\emptyset \end{bmatrix} = \begin{bmatrix} sin\theta cos\emptyset & sin\theta sin\emptyset & cos\theta \\ cos\theta cos\emptyset & cos\theta sin\emptyset & -sin\theta \\ -sin\emptyset & cos\emptyset & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Spherical to Rectangular:

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} sin\theta cos\emptyset & cos\theta cos\emptyset & -sin\emptyset \\ sin\theta sin\emptyset & cos\theta sin\emptyset & cos\emptyset \\ cos\theta & -sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\emptyset \end{bmatrix}$$

Cylindrical to Spherical:

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\emptyset \end{bmatrix} = \begin{bmatrix} sin\theta & 0 & cos\theta \\ cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\emptyset \\ A_z \end{bmatrix}$$

Some Physical Constants

Permittivity of free space:

$$\varepsilon_0 = 8.854 \times 10^{-12}\ F/m$$

Permeability of free space:

$$\mu_0 = 4\pi \times 10^{-7}\ H/m$$

Velocity of light in free space:

$$c = 2.988 \times 10^{8}\ m/s$$

Electron charge:

$$e = 1.6 \times 10^{-19}\ C$$

Intrinsic impedance of free space:

$$\eta = 377\ \Omega$$

Vector Differential Operators

Rectangular Coordinates:

$$\vec{\nabla}V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$$

$$\vec{\nabla}.\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical Coordinates:

$$\vec{\nabla}V = \frac{\partial V}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial V}{\partial \emptyset}\hat{\emptyset} + \frac{\partial V}{\partial z}\hat{z}$$

$$\vec{\nabla}.\vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\emptyset}{\partial \emptyset} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla} \times \vec{A} = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\emptyset} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \emptyset} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\emptyset & A_z \end{vmatrix}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \emptyset^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical Coordinates:

$$\vec{\nabla}V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{rsin\theta}\frac{\partial V}{\partial \emptyset}\hat{\emptyset}$$

$$\vec{\nabla}.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{rsin\theta}\frac{\partial}{\partial \theta}(A_\theta sin\theta) + \frac{1}{rsin\theta}\frac{\partial A_\emptyset}{\partial \emptyset}$$

$$\vec{\nabla} \times \vec{A} = \frac{1}{r^2 sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & rsin\theta\hat{\emptyset} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \emptyset} \\ A_r & rA_\theta & rsin\theta A_\emptyset \end{vmatrix}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 V}{\partial \emptyset^2}$$

APPENDIX 2: WAVEGUIDE PARAMETERS

Rectangular Waveguide

Quantity	TE_{mn} Mode	TM_{mn} Mode
k	$\omega\sqrt{\mu\epsilon}$	$\omega\sqrt{\mu\epsilon}$
k_c	$\sqrt{(m\pi/a)^2+(n\pi/b)^2}$	$\sqrt{(m\pi/a)^2+(n\pi/b)^2}$
β	$\sqrt{k^2-k_c^2}$	$\sqrt{k^2-k_c^2}$
λ_c	$\frac{2\pi}{k_c}$	$\frac{2\pi}{k_c}$
λ_g	$\frac{2\pi}{\beta}$	$\frac{2\pi}{\beta}$
v_p	$\frac{\omega}{\beta}$	$\frac{\omega}{\beta}$
α_d	$\frac{k^2\tan\delta}{2\beta}$	$\frac{k^2\tan\delta}{2\beta}$
Z	$Z_{TE}=\frac{k\eta}{\beta}$	$Z_{TM}=\frac{\beta\eta}{k}$

$$\alpha_c=\frac{P_l}{2P_{10}}=\frac{2\pi^2R_s\left(b+\frac{a}{2}+\frac{\beta^2a^3}{2\pi^2}\right)}{\omega\mu a^3b\beta}=\frac{R_s}{a^3b\beta k\eta}(2b\pi^2+a^3k^2);\ R_s=\sqrt{\frac{\omega\mu_0}{2\sigma}}$$

Circular Waveguide:

Quantity	TE_{nm} Mode	TM_{nm} Mode
k	$\omega\sqrt{\mu\epsilon}$	$\omega\sqrt{\mu\epsilon}$
k_c	$\frac{p'_{nm}}{a}$	$\frac{p_{nm}}{a}$
β	$\sqrt{k^2-k_c^2}$	$\sqrt{k^2-k_c^2}$
λ_c	$\frac{2\pi}{k_c}$	$\frac{2\pi}{k_c}$
λ_g	$\frac{2\pi}{\beta}$	$\frac{2\pi}{\beta}$
v_p	$\frac{\omega}{\beta}$	$\frac{\omega}{\beta}$
α_d	$\frac{k^2\tan\delta}{2\beta}$	$\frac{k^2\tan\delta}{2\beta}$
Z	$Z_{TE}=\frac{k\eta}{\beta}$	$Z_{TM}=\frac{\beta\eta}{k}$

n	p'_{n1}	p'_{n2}	p'_{n3}
0	3.832	7.016	10.174
1	1.841	5.331	8.536
2	3.054	6.706	9.970

n	p_{n1}	p_{n2}	p_{n3}
0	2.405	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.620

$$\alpha_c = \frac{P_l}{2P_0} = \frac{R_s(k_c^4 a^2+\beta^2)}{\eta k\beta a(p_{11}'^2-1)} = \frac{R_s}{ak\eta\beta}\left(k_c^2 + \frac{k^2}{p_{11}'^2-1}\right);\ R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}}$$

APPENDIX 3: BINOMIAL AND CHEBYSHEV MATCHING TRANSFORMERS

Reflection coefficient at the nth section:

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

For Binomial transformer:

$$\mathrm{A} = 2^{-\mathrm{N}}\frac{\mathrm{Z_L}-\mathrm{Z_0}}{\mathrm{Z_L}+\mathrm{Z_0}};\ C_n^N = \frac{N!}{(N-n)!n!};\qquad \Gamma_n = AC_n^N$$

$$\ln\frac{Z_{n+1}}{Z_n} \simeq 2\Gamma_n = 2AC_n^N = 2(2^{-N})\frac{Z_L - Z_0}{Z_L + Z_0}C_n^N \simeq 2^{-N}C_n^N \ln\frac{Z_L}{Z_0}$$

Defining Γ_m, as maximum reflection coefficient that can be tolerated:

$$\frac{1}{\Gamma_\mathrm{m}^2} = 1 + \left(\frac{2\sqrt{\mathrm{Z_o Z_L}}}{\mathrm{Z_L} - \mathrm{Z_o}}\sec\theta_\mathrm{m}\right)^2,\quad \cos\theta_\mathrm{m} = \frac{\Gamma_\mathrm{m}}{\sqrt{1-\Gamma_\mathrm{m}^2}}\left(\frac{2\sqrt{\mathrm{Z_o Z_L}}}{|\mathrm{Z_L} - \mathrm{Z_o}|}\right)$$

Fractional bandwidth:

$$\frac{\Delta\mathrm{f}}{\mathrm{f_o}} = \frac{2(\mathrm{f_o} - \mathrm{f_m})}{\mathrm{f_o}} = 2 - \frac{2\mathrm{f_m}}{\mathrm{f_o}} = 2 - \frac{4\theta_\mathrm{m}}{\pi} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{1}{2}\left(\frac{\Gamma_\mathrm{m}}{|A|}\right)^{\frac{1}{N}}\right]$$

First four polynomials for the Chebyshev transformer design:

$$T_1(x) = x;\ T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x;\ T_4(x) = 8x^4 - 8x^2 + 1$$

$$T_1(sec\theta_m cos\theta) = sec\theta_m cos\theta;\ T_2(sec\theta_m cos\theta) = sec^2\theta_m(1 + cos2\theta) - 1$$

$$T_3(sec\theta_m cos\theta) = sec^3\theta_m(cos3\theta + 3cos\theta) - 3sec\theta_m cos\theta$$

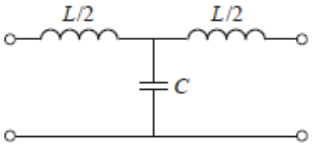
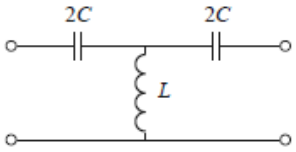
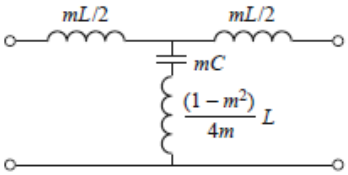
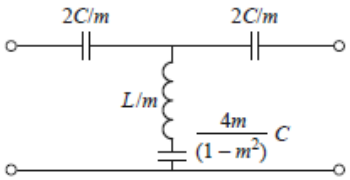
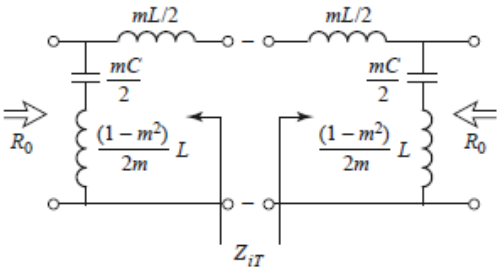
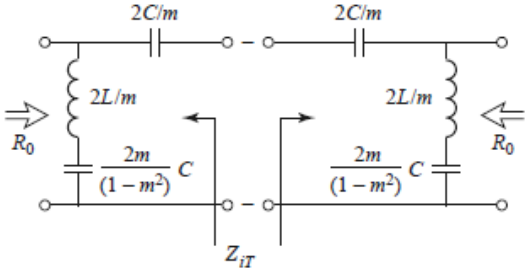
$$T_4(sec\theta_m cos\theta) = sec^4\theta_m(cos4\theta + 4cos2\theta + 3) - 4sec^2\theta_m(1 + cos2\theta) + 1$$

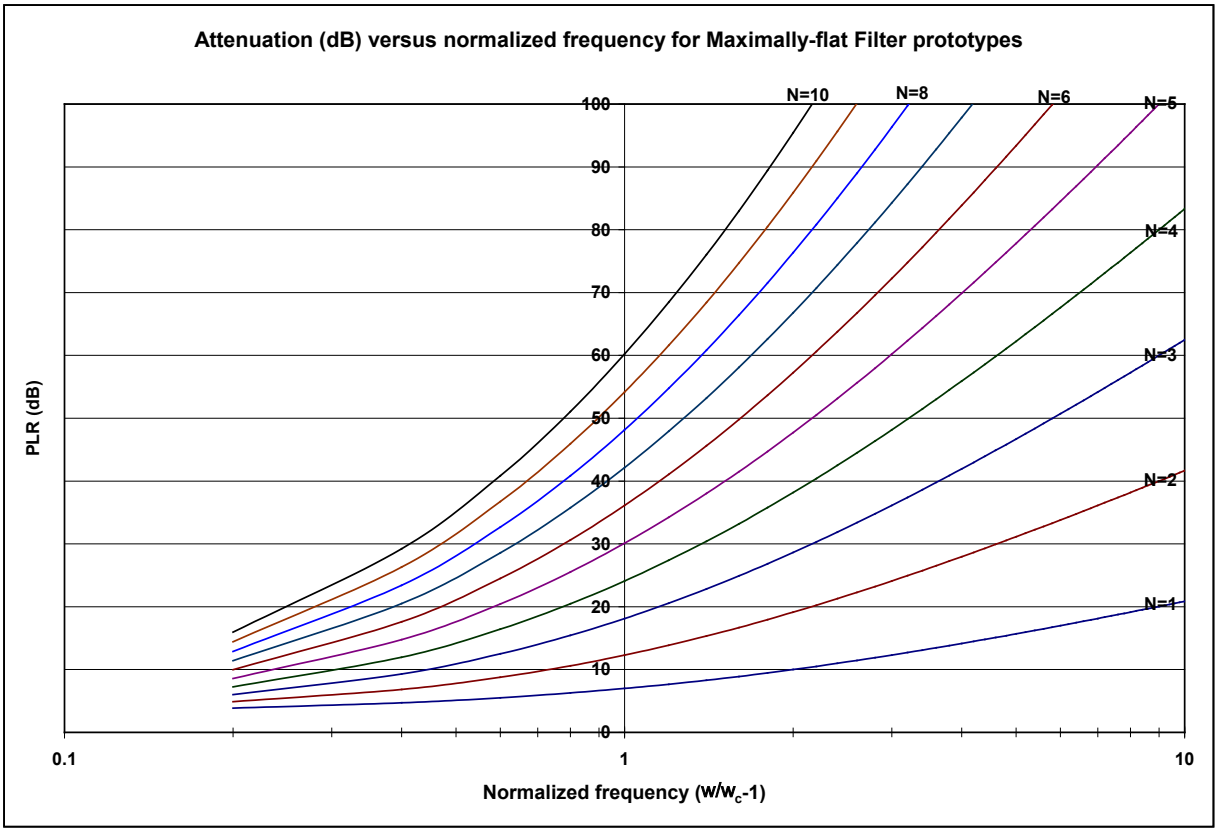
For θ_m:

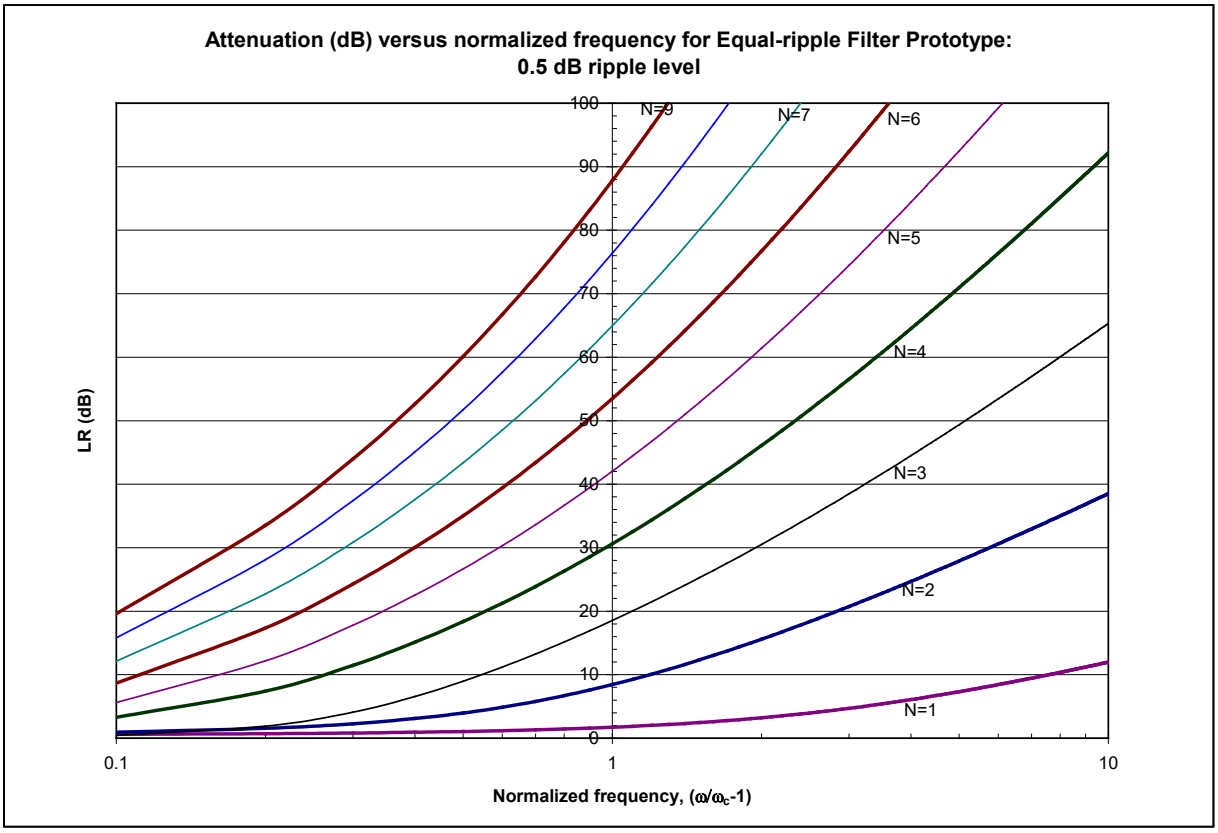
$$\sec\theta_m = \cosh\left[\frac{1}{N}\cosh^{-1}\left(\frac{1}{\Gamma_m}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|\right)\right]$$

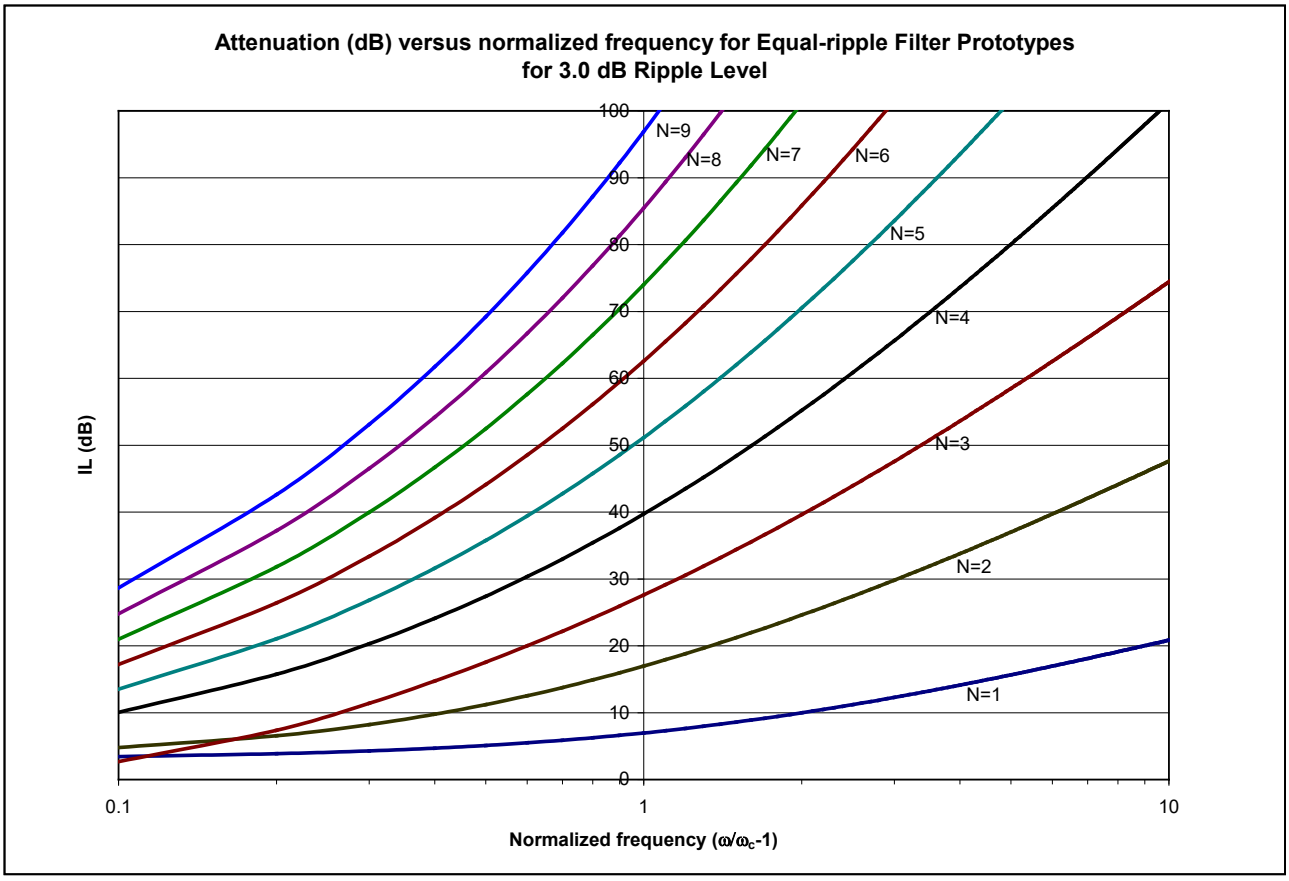
APPENDIX 4: MICROWAVE FILTERS

Table: Composite filter design

Low-Pass	High-Pass
Constant-k T section  $R_0 = \sqrt{L/C}$ $\quad L = 2R_0/\omega_c$ $\omega_c = 2/\sqrt{LC}$ $\quad C = 2/\omega_c R_0$	**Constant-k T section**  $R_0 = \sqrt{L/C}$ $\quad L = R_0/2\omega_c$ $\omega_c = 1/2\sqrt{LC}$ $\quad C = 1/2\omega_c R_0$
m-derived T section  L, C Same as constant-k section $m = \begin{cases} \sqrt{1-(\omega_c/\omega_\infty)^2} & \text{for sharp-cutoff} \\ 0.6 & \text{for matching} \end{cases}$	**m-derived T section**  L, C Same as constant-k section $m = \begin{cases} \sqrt{1-(\omega_\infty/\omega_c)^2} & \text{for sharp-cutoff} \\ 0.6 & \text{for matching} \end{cases}$
Bisected-π matching section  	**Bisected-π matching section** 


Attenuation (dB) versus normalized frequency for Maximally-flat Filter prototypes
PLR (dB)
100
90
80
70
60
50
40
30
20
10
0
N=10
N=8
N=6
N=5
N=4
N=3
N=2
N=1
0.1
1
10
Normalized frequency (w/w$_c$-1)


Attenuation (dB) versus normalized frequency for Equal-ripple Filter Prototype:
0.5 dB ripple level
LR (dB)
100
90
80
70
60
50
40
30
20
10
0
N=9
N=7
N=6
N=5
N=4
N=3
N=2
N=1
0.1
1
10
Normalized frequency, (ω/ω$_c$-1)



n	$n-th\ order\ Butterworth\ Polynomial$
1	$(s+1)$
2	$(s^2+\sqrt{2}s+1)$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.765s+1)(s^2+1.848s+1)$
5	$(s+1)(s^2+0.618s+1)(s^2+1.618s+1)$
6	$(s^2+0.518s+1)(s^2+\sqrt{2}s+1)(s^2+1.932s+1)$
7	$(s+1)(s^2+0.445s+1)(s^2+1.247s+1)(s^2+1.802s+1)$
8	$(s^2+0.390s+1)(s^2+1.111s+1)(s^2+1.6663s+1)(s^2+1.962s+1)$

Table: Element values for maximally flat low-pass filter prototypes ($g_0 = 1, \omega_c = 1, N = 1\ to\ 10$)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	**2.0000**	**1.0000**									
2	**1.4142**	**1.4142**	**1.0000**								
3	**1.0000**	**2.0000**	**1.0000**	**1.0000**							
4	**0.7654**	**1.8478**	**1.8478**	**0.7654**	**1.0000**						
5	**0.6180**	**1.6180**	**2.0000**	**1.6180**	**0.6180**	**1.0000**					
6	**0.5176**	**1.4142**	**1.9318**	**1.9318**	**1.4142**	**0.5176**	**1.0000**				
7	**0.4450**	**1.2470**	**1.8019**	**2.0000**	**1.8019**	**1.2470**	**0.4450**	**1.0000**			
8	**0.3902**	**1.1111**	**1.6629**	**1.9615**	**1.9615**	**1.6629**	**1.1111**	**0.3902**	**1.0000**		
9	**0.3473**	**1.0000**	**1.5321**	**1.8794**	**2.0000**	**1.8794**	**1.5321**	**1.0000**	**0.3473**	**1.0000**	
10	**0.3129**	**0.9080**	**1.4142**	**1.7820**	**1.9754**	**1.9754**	**1.7820**	**1.4142**	**0.9080**	**0.3129**	**1.0000**

Table: Element values for equal-ripple low-pass filter prototypes ($g_0 = 1, \omega_c = 1, N = 1\ to\ 10$, 0.5 dB ripple)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	**0.6986**	**1.0000**									
2	**1.4029**	**0.7071**	**1.9841**								
3	**1.5963**	**1.0967**	**1.5963**	**1.0000**							
4	**1.6703**	**1.1926**	**2.3661**	**0.8419**	**1.9841**						
5	**1.7058**	**1.2296**	**2.5408**	**1.2296**	**1.7058**	**1.0000**					
6	**1.7254**	**1.2479**	**2.6064**	**1.3137**	**2.4758**	**0.8696**	**1.9841**				
7	**1.7372**	**1.2583**	**2.6381**	**1.3444**	**2.6381**	**1.2583**	**1.7372**	**1.0000**			
8	**1.7451**	**1.2647**	**2.6564**	**1.3590**	**2.6964**	**1.3389**	**2.5093**	**0.8796**	**1.9841**		
9	**1.7504**	**1.2690**	**2.6678**	**1.3673**	**2.7239**	**1.3673**	**2.6678**	**1.2690**	**1.7504**	**1.0000**	
10	**1.7543**	**1.2721**	**2.6754**	**1.3725**	**2.7392**	**1.3806**	**2.7231**	**1.3485**	**2.5239**	**0.8842**	**1.9841**

Table: Element values for equal-ripple low-pass filter prototypes ($g_0 = 1, \omega_c = 1, N = 1\ to\ 10$, 3 dB ripple)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Filter transformation:

$$\omega \leftarrow \frac{\omega_0}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

where $\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$

Table: Prototype filter transformation $\left(\Delta = \frac{\omega_2 - \omega_1}{\omega_0}\right)$

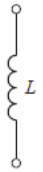
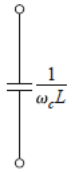
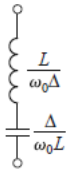
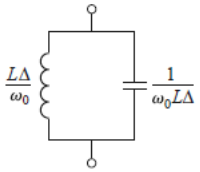
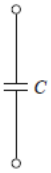
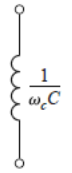
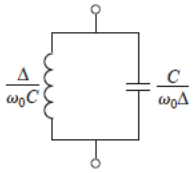
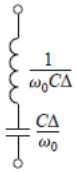
Low-pass	High-pass	Bandpass	Bandstop
			
			

Table: Coupled line bandpass filter design

$Z_0 J_1 = \sqrt{\frac{\pi\Delta}{2g_1}}$, $Z_0 J_n = \frac{\pi\Delta}{2\sqrt{g_{n-1}g_n}}$ for $n = 2, 3, \ldots, N$, $Z_0 J_{N+1} = \sqrt{\frac{\pi\Delta}{2g_N g_{N+1}}}$.	$Z_{0e} = Z_0[1 + JZ_0 + (JZ_0)^2]$ $Z_{0o} = Z_0[1 - JZ_0 + (JZ_0)^2]$

APPENDIX 5: STRIPLINE

$$Z_0 = \frac{30\Pi}{\sqrt{\varepsilon_r}}\left(\frac{b}{w_e + 0.441}\right)\Omega;\quad \frac{w_e}{b} = \frac{w}{b} - \left\{\begin{array}{l} 0 \; for \; \frac{w}{b} > 0.35 \\ (0.35 - \frac{w}{b})^2 \; for \; \frac{w}{b} < 0.35 \end{array}\right\},$$

where w is width of conductor, b is substrate depth.

$$\frac{W}{b} = \begin{cases} x & \text{for } \sqrt{\epsilon_r} Z_0 < 120\ \Omega \\ 0.85 - \sqrt{0.6 - x} & \text{for } \sqrt{\epsilon_r} Z_0 > 120\ \Omega, \end{cases}$$

where

$$x = \frac{30\pi}{\sqrt{\epsilon_r} Z_0} - 0.441$$

The dielectric attenuation constant is given by:

$$\alpha_d = \frac{k tan\delta}{2}$$

The conductor attenuation constant is given by:

$$\alpha_c = \left\{\begin{array}{l} \frac{2.7X10^{-3} R_s \varepsilon_r Z_0}{30\Pi(b-t)} A \quad for \quad \sqrt{\varepsilon_r} Z_0 < 120 \quad (Neper/m) \\ \frac{0.16 R_s}{Z_0 b} B \quad for \quad \sqrt{\varepsilon_r} Z_0 > 120 \quad (Neper/m) \end{array}\right\}$$

$$A = \left[1 + \frac{2w}{b-t} + \frac{1}{\Pi}\frac{b+t}{b-t}\ln\left(\frac{2b-t}{t}\right)\right]$$

$$B = \left[1 + \frac{b}{(0.5w + 0.7t)}(0.5 + \frac{0.414t}{w} + \frac{1}{2\Pi}\ln(\frac{4\Pi w}{t}))\right]$$

R_s is the conductor surface resistance.

APPENDIX 6: MICROSTRIP LINE

Effective relative permittivity: $\varepsilon_{reff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2\sqrt{1+12h/w}}$

Characteristic impedance: $Z_0 = \begin{cases} \frac{60}{\sqrt{\varepsilon_{reff}}} ln\left(\frac{8h}{w} + \frac{w}{h}\right) & w/h \le 1 \\ \frac{1}{\sqrt{\varepsilon_{reff}}} \frac{120\pi}{\left[\frac{w}{h}+1.393+0.667ln\left(\frac{w}{h}+1.444\right)\right]} & w/h \ge 1 \end{cases}$

$$\frac{w}{h} = \begin{cases} \dfrac{8e^A}{e^{2A}-2} & w/h \le 2 \\ \dfrac{2}{\pi}\left[B - 1 - \ln(2B-1) + \dfrac{\varepsilon_r - 1}{2\varepsilon_r}\left\{\ln(B-1) + 0.39 - \dfrac{0.61}{\varepsilon_r}\right\}\right] & w/h \ge 2 \end{cases}$$

Where: $A = \frac{Z_0}{60}\sqrt{\frac{\varepsilon_r-1}{2}} + \frac{\varepsilon_r-1}{\varepsilon_r+1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right)$; $B = \frac{60\pi^2}{Z_0\sqrt{\varepsilon_r}}$

The dielectric attenuation constant is given by:

$$\alpha_d = \frac{k_0\varepsilon_r(\varepsilon_{reff}-1)tan\delta}{2\sqrt{\varepsilon_{reff}}(\varepsilon_r-1)}$$

The conductor attenuation constant is given by:

$$\alpha_c = \frac{1}{Z_0 w}\sqrt{\frac{\omega\mu_0}{2\sigma}}$$

APPENDIX 7: MICROSTRIP ANTENNAS

Effective dielectric constant:

$$\varepsilon_{reff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left[1 + 12\frac{h}{w}\right]^{-\frac{1}{2}} \quad \frac{W}{h} > 1$$

$$\frac{\Delta L}{h} = 0.412\frac{(\varepsilon_{\text{reff}}+0.3)(\frac{\text{W}}{\text{h}}+0.264)}{(\varepsilon_{\text{reff}}-0.258)(\frac{\text{W}}{\text{h}}+0.8)}$$

Effective Patch Length: $L_{eff} = \frac{\lambda_0}{2} = \frac{v}{2f^{(001)}} = \frac{c}{2f_r{}^{(001)}\sqrt{\varepsilon_{reff}}}$

Patch width: $W = \frac{1}{2f_r\sqrt{\mu_0\varepsilon_0}}\sqrt{\frac{2}{\varepsilon_r+1}} = \frac{c}{2f_r}\sqrt{\frac{2}{\varepsilon_r+1}}$

Resonant frequency of circular patch: $(f_r)_{mn0} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\left(\frac{X'_{mn}}{a}\right)$

Effective radius: $a_e = a\left[1 + \frac{2h}{\pi a\varepsilon_r}\left\{ln\left(\frac{\pi a}{2h}\right) + 1.7726\right\}\right]^{0.5}$

Actual radius: $a = \frac{F}{\left[1+\frac{2h}{\pi\varepsilon_r F}\left\{ln\left(\frac{\pi F}{2h}\right)+1.7726\right\}\right]^{0.5}}$, where $F = \frac{8.791\times10^9}{f_r\sqrt{\varepsilon_r}}$, h and a are in cm.